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as the proper source from which phosphorus, on the large scale, is to be procured. On the small scale, shavings of hartshorn were stated to be more convenient.

Estimates were then given of the quantities of the materials to be employed under various circumstances, with their cost, and the mode of manipulation. Directions were given for recovering the acetic acid disengaged in the process, and reconverting it into acetate of lead for future precipitations of bone solutions. An easy method of drying and reducing the volume of the phosphate of lead obtained was described. The paper concluded with two formulæ for obtaining phosphorus, founded on the facts stated, which it was conceived reduce the trouble and cost of preparing that article to the lowest scale of which it is susceptible.

Mr. H. L. Renny read a paper on the effects of moisture as affecting the barometric measurement of heights.

"Whereas Dr. Apjohn has inserted in a note of a paper read by him before the Academy, and published in vol. ii. of the Proceedings of the Academy (1840–1844), at page 565, an expression for the correction due to the hygrometric state of the atmosphere, in the formulæ for the measurement of heights by the barometer, which expression, as the note states, was furnished to Dr. Apjohn by myself, I hope I do not request unnecessarily the attention of the Academy to the process by which I obtained the said formula.

$$\left.\begin{array}{l}\text{Let } p \text{ be pressure,}\\ \quad f \text{ be the force of aqueous vapour,}\end{array}\right\}\text{at the lower station.}$$

$$\left.\begin{array}{l}p' \text{ be pressure,}\\ f' \text{ be the force of aqueous vapour,}\end{array}\right\}\text{at the upper station.}$$

$$\left.\begin{array}{l}\pi \text{ be pressure,}\\ F \text{ be the force of aqueous vapour,}\end{array}\right\}\begin{array}{l}\text{at any station whatever, } \pi \text{ and } F \text{ being,}\\ \text{of course, variable.}\end{array}$$

n be a number extremely great.

δ be a quantity indefinitely small.

Let r be ratio of a geometric series.

M be (= 0 . 43494, &c.) modulus of common logarithms.

v be hypothetic distance between stations, upon supposition that the atmosphere be perfectly free from aqueous vapour.

v' be actual distance between stations, taking, of course, into consideration the hygrometric state of the atmosphere.

Now let us suppose the actual distance between the stations ($= v'$) to be divided into an extremely great number of equally thin parts or strata, then $\frac{v'}{n} =$ actual thickness of each equal stratum of air; also, $\frac{v'}{n} \times \frac{\pi - F}{\pi}$ is the general expression for the hypothetic thickness of any stratum, upon supposition that the atmosphere be perfectly free from aqueous vapour.*

" Now, adopting a notation (similar to that employed for the upper station), relative to the pressure and force of vapour, for the successive strata of air, descending from the upper to the lower station, we have for expression of the hypothetic thicknesses of the various strata, upon supposition that the air be perfectly dry,

$$\frac{v'}{n} \times \frac{p'-f'}{p'}; \quad \frac{v'}{n} \times \frac{p''-f''}{p''}; \quad \frac{v'}{n} \times \frac{p'''-f'''}{p'''}; \quad \frac{v'}{n} \times \frac{p''''-f''''}{p''''}; \text{ \&c.}$$

Now, the whole being equal to the sum of its parts,

v (– the hypothetic distance between the stations) $= \frac{v'}{n} \times \frac{p'-f'}{p'}$

$$+ \frac{v'}{n} \times \frac{p''-f''}{p''} + \frac{v'}{n} \times \frac{p'''-f'''}{p'''}; + \text{\&c. \&c.}$$

that is,

$$v = \frac{v'}{n} \times \left\{ \overline{1 - \frac{f'}{p'}} + \overline{1 - \frac{f''}{p''}} + \overline{1 - \frac{f'''}{p'''}} + \overline{1 - \frac{f''''}{p''''}} + \text{\&c.} \right\}$$

But p', p'', p''', p'''', &c., form a geometric series, according to

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* *Vide* paper by Dr. Apjohn, Proceedings Royal Irish Academy, vol. ii. p. 563 ; or p. 105, same volume.

the well-known principle of barometric measurements; and by consulting the table of forces of aqueous vapour, in the appendix to Turner's Chemistry,* I find that the forces of vapour form (*quam proxime*) a geometric series, when the degrees of temperature form an arithmetic one. Let us, therefore, take the geometric mean of the forces of aqueous vapour at the stations $= \sqrt{(f \times f')}$, and indicating this quantity F', and instead of the variable forces of vapour in the last equation, let us employ the quantity F' (which will be practically sufficiently accurate so long as the correction for the temperature of the air, as shown by the detached thermometers, continues, as at present, so liable to error), we shall change our fundamental equation, as given above, into

$$v = \frac{v'}{n} . \left\{ \overline{1 - \frac{F'}{p'}} + \overline{1 - \frac{F'}{rp'}} + \overline{1 - \frac{F'}{r^2p'}} + \overline{1 - \frac{F'}{r^3p'}} + \&c. \, . \, . \, \overline{1 - \frac{F'}{r^{n-1}p'}} \right\},$$

or

$$v = \frac{v'}{n} \left\{ n - F'\left(\frac{1}{p'} + \frac{1}{rp'} + \frac{1}{r^2p'} + \frac{1}{r^3p'} + \&c. \, . \, . \, \frac{1}{r^{n-1}p'} \right) \right\}.$$

Summing the geometric series of the right hand of the equation last obtained, and modifying somewhat the rest of it, we have

$$v = v' \left\{ 1 - F' . \frac{1}{n} . \left(\frac{\frac{1}{p'} - \frac{1}{r^n p'}}{1 - \frac{1}{r}} \right) \right\}. \qquad \text{(A)}$$

But $r^{n-1} p' = p$; eliminating r from equation (A), by means of this last equation, we have

$$v = v' \left\{ 1 - F', \frac{\frac{1}{p'} - \frac{1}{p'}\left(\frac{p'}{p}\right)^{\frac{n}{n-1}}}{n\left(1 - \left(\frac{p'}{p}\right)^{\frac{1}{n-1}}\right)} \right\}. \qquad \text{(B)}$$

Let us now seek the limit of the right hand side of equa-

* Seventh Edit. pp. 1248–49.

tion (B). By changing (n), a quantity extremely great, into (n'), a quantity indefinitely great, we have $n' = (n' - 1)$; also, $\frac{n'}{n'-1} = 1$. Moreover, p' being less than p, $\frac{p'}{p}$ is a fraction, and the limit of $\left(\frac{p'}{p}\right)^{\frac{1}{n'}} =$ unity, which is expressed in algebraic language thus,

$$\left(\frac{p'}{p}\right)^{\frac{1}{n-1}} = 1 - \delta;$$

consequently

$$n'.\left\{1 - \left(\frac{p'}{p}\right)^{\frac{1}{n'-1}}\right\} \text{ (when we take its limits)} = n'(1 - \overline{1-\delta}) = n'\delta.$$

Now, whereas

$$\left(\frac{p'}{p}\right)^{\frac{1}{n'}} = \overline{1-\delta};\ \log\left(\frac{p'}{p}\right) \times \frac{1}{n'} = \log.\overline{1-\delta} = -M\left\{\frac{\delta}{1} + \frac{\delta^2}{2} + \frac{\delta^3}{3} + \&c.\right\};$$

or
$$n' = \frac{\log\frac{p'}{p}}{-M\left(\frac{\delta}{1} + \frac{\delta^2}{2} + \frac{\delta^3}{3} + \&c.\right)};$$

or
$$n'\delta = \frac{\log\frac{p'}{p}}{-M - M\left(\frac{\delta}{2} + \frac{\delta^2}{3} + \frac{\delta^3}{4} + \&c.\right)};$$

or
$$n'\delta = \frac{\log.\frac{p}{p'}}{M + M\left(\frac{\delta}{2} + \frac{\delta^2}{3} + \frac{\delta^3}{4} + \&c.\right)}.$$

Taking limits, by omitting δ and its powers in the right hand side of this last equation, we have the limit of $n'\delta$, or

$$n'\left\{1 - \left(\frac{p'}{p}\right)^{\frac{1}{n-1}}\right\} = \frac{\log.\frac{p}{p'}}{M};$$

substituting this value in the right hand side of equation (B), also simplifying the numerator, we have

$$v = v' \cdot \left\{ 1 - F' \frac{\left(\frac{1}{p'} - \frac{1}{p}\right)}{\frac{\log \frac{p}{p'}}{M}} \right\};$$

therefore
$$v' = v \times \frac{1}{1 - F' \cdot \frac{\frac{1}{p'} - \frac{1}{p}}{\frac{1}{M} \cdot \log \frac{p}{p'}}}.$$

Modifying the right hand side of this last equation, by dividing each term by the coefficient of F', we have

$$v' = v \cdot \frac{\frac{\frac{1}{M} \log \frac{p}{p'}}{\frac{1}{p'} - \frac{1}{p}}}{\frac{\frac{1}{M} \log \frac{p}{p'}}{\frac{1}{p'} - \frac{1}{p}} - F'}.$$

Let
$$\frac{\frac{1}{M} \cdot \log \frac{p}{p'}}{\frac{1}{p'} - \frac{1}{p}} \text{ be } P;$$

then $v' = v \times \frac{P}{P - F'}$; being the formula given in note, Proceedings of Royal Irish Academy, vol. ii. p. 565; F' being $\sqrt{(f \times f')}$, or the geometric mean of forces of aqueous vapour.

N. B.—The approximate expression given by Dr. Apjohn,* viz.,

$$v' = v \times \frac{\sqrt{\{(p - f') \times (p' - F'')\}}}{\sqrt{\{(p - f'') \times (p' - F'')\}} - \frac{1}{2}(f'' + F'')},$$

in which Dr. Apjohn employs the geometric mean of pressures minus the forces of aqueous vapour, instead of P, the more correct expression, will answer very well indeed for hills only 1000 feet high, and under that height. In fact, for hills of such height, Dr. Apjohn's formula is astonishingly close to the more correct expression. But for hills 2000 feet high and upwards, Dr. Apjohn's approximate formula fails, inasmuch as the error varies from 10 to 20 per cent. of the correction due to the hygrometric state of the air. Now, as Dr. Apjohn justly observes† that the correction due to the hygrometric state of the air amounts to at least 30 feet in hills 2000 feet high, the error of his formula will vary from 3 to 6 feet, according to the greater or smaller quantity of watery vapour in the atmosphere. Indeed in hills of 1000 feet high and less, instead of the geometric mean of pressures minus the forces of aqueous vapour, we may employ the arithmetic mean of pressures with perfect practical safety, viz.:

$$v' = v \times \frac{\frac{1}{2}(p + p')}{\frac{1}{2}(p + p') - \frac{1}{2}(f + f')} = v \times \frac{(p + p')}{(p + p') - (f + f')}.$$

N. B.—The formula given by Mr. Renny to Dr. Apjohn, viz.,

$$v' = v \times \frac{P}{P - F},$$

P being

$$\frac{\frac{1}{M}\log\frac{p}{p'}}{\left(\frac{1}{p'} - \frac{1}{p}\right)},$$

is not rigidly or mathematically correct; because Mr. Renny's

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* Proceedings Royal Irish Academy, vol. ii. p. 563.

† Ibid. vol. ii. p. 564.

fundamental equation,

$$v = \frac{v'}{n} \cdot \left\{ n - F' \left(\frac{1}{p'} + \frac{1}{rp'} + \frac{1}{r^2 p'} + \frac{1}{r^3 p'} + \&c. \frac{1}{r^{n-1} p'} \right) \cdot \right\},$$

supposes that the hypothetic thicknesses of the strata of air are equal, which is not true, for they vary as $\frac{\pi - F}{\pi}$. Considering, however, that until the law of variation of temperature of the atmosphere between the stations be determinately known (which will, perhaps, never take place), the barometric formula for heights can only be approximate, it is lawful to employ the said formula as closely approximate, until, however, a more correct one be obtained.

The mathematic error thus noticed escaped Mr. Renny's attention when, six years ago, he gave the formula to Dr. Apjohn. Mr. Renny hopes, at no distant period, to obtain a formula absolutely correct, if not by series, by the integral calculus.

The Secretary of Council read the following communication from Sir William Rowan Hamilton, on an equation of the ellipsoid.

" A remark of your's, recently made, respecting the form in which I first gave to the Academy, in December, 1845, an equation of the ellipsoid by quaternions,—namely, that this form involved only *one* asymptote of the focal hyperbola,—has induced me to examine, simplify, and extend, since I last saw you, some manuscript results of mine on that subject; and the following new form of the equation, which seems to meet your requisitions, may, perhaps, be shewn to the Academy to-night. This new form is the following :

$$\mathrm{TV}\,\frac{\eta\rho - \rho\theta}{\mathrm{U}(\eta - \theta)} = \theta^2 - \eta^2. \qquad (1)$$

" The constant vectors η and θ are in the directions of the two asymptotes required ; their symbolic sum, $\eta + \theta$, is the vector of